DETROIT COUNTRY DAY SCHOOL'S

THIRTY-FOURTH ANNUAL

MATHEMATICS INVITATIONAL

FEBRUARY 6, 2010

INSTRUCTIONS

1. Please print your name, address, grade, and school on the scan-tron. Be sure to mark each answer in the appropriate space on the scan-tron sheet. If you do not have a #2 pencil, please ask for one before the test starts.

2. Do as many of the 30 problems as you can in the 45 minutes allotted.

3. The test is designed to challenge you. Do not get discouraged if the problems do not seem easy. You are here because you are a good mathematician.

4. Most of the problems require some computation. Work out your answers on scrap paper before selecting your answer. You can write on the paper containing the test questions and you should mark your answers to check yourself later.

5. There are certain questions that were designated as tiebreakers. In case of a tie, in the total score, the student getting the greatest number of tiebreakers correct will be the winner. The tie-breaker questions are 26-30.

6. Only the answers marked on the scan-tron sheet will be used to determine your score. The scan-tron machine is very sensitive and reads all the marks. If you erase, do it thoroughly. The answers to the five tie-breakers, questions 26-30, should be recorded at the designated locations on the scan-tron.

7. The people supervising this test are not permitted to explain the meaning of any question. If you have questions concerning the instructions, ask them now.

8. You are not allowed to use a calculator during this part of the competition.

9. GOOD LUCK !!!!

1) $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$ of 4^3 is $\frac{1}{2}$ of what number?.

    a) ½   b) 64   c) 8   d) 16   e) 4

2) What is the product of any non-zero number and twice it's reciprocal?

    a) 0.5   b) 1   c) 1.5   d) 2   e) can't be determined

3) Dave, Walter, and Joe are members of the gym and always workout from 6am- 8am when they go. They often run into each other and have become friends over the 4 years that they all have been members. Dave goes to the gym every eight days, Walter goes to the gym every four days, and Joe goes every ten days. If today is Saturday and all three friends were working out together, what day of the week will be the next time that Joe and Walter will work out without seeing Dave?

    a) Monday   b) Friday   c) Sunday   d) Thursday   e) Saturday

4) A rectangle has its length increased so that it is 1/5 larger than before. Its width is decreased so that it is 1/10 smaller than before. This makes the area of the rectangle:

    a) 6.5% greater   b) 8% greater   c) 10% greater   d) 12.5% greater e) 30% greater

5) 5 even numbers are removed from the first 25 positive whole numbers. What percent of the remaining numbers are even?

    a)   20%   b)   25%   c)   28%   d)   30%   e)   35%

6) A carpenter agrees to work for $2 per day and to give back $3 every day he does not work. At the end of 30 days he finds he has given back as much as he earned. How many days did he work?

    a) 18 days   b) 20 days   c) 21 days   d) 24 days   e) 25 days

7) Three friends head off in different directions, each to visit a different Michigan landmark. After returning home, they determined that they had traveled a total of 2652 miles. If their individual miles traveled were in the ratio of 2:3:5, what was the greatest individual distance (in miles) covered?

a) 265.2   b) 530.4   c) 795.6   d) 1326   e) 1856.4

8) Your school band has been asked to march in the Mackinaw Bridge Parade on Labor Day Weekend. Your band director found that he can arrange all the members in the band in rows of exactly 6, exactly 7, or exactly 8 with no one left over. What is the least number of members that could be in your school band?

a) 42   b) 56   c) 168   d) 248   e) 310

9) For the equation $y = 3 - |x - 2|$, let x have the values -100, -8.12, 0, $2\frac{1}{3}$, and 25. For which of the given values for x will y have the largest value.

a) -100   b) -8.12   c) 0   d) $2\frac{1}{3}$ e) 25

10) If two six-sided fair dice are rolled and each side is numbered 1 through 6, what is the probability that the product of the two numbers will be odd?

a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{7}{36}$ d) $\frac{1}{2}$ e) $\frac{7}{12}$
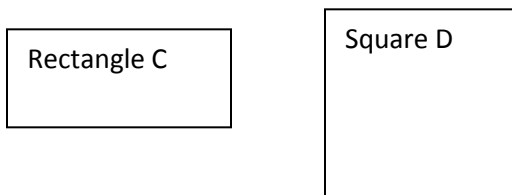
11) If a ψ b means $\frac{(a+b)(a-b)}{ab-b}$, then find 0.5 ψ 0.25.

a) 1.5   b) 0.75   c) 1   d) -0.75   e) -1.5

12) Rectangle A and Square B each have an area of 144 sq. cm. The width of Rectangle A is 4 cm less than the width of Square B.

Square B
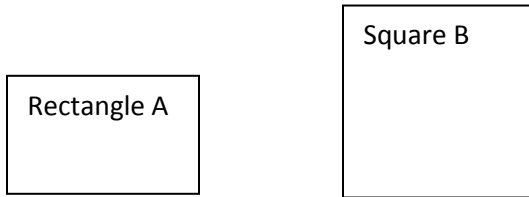
Rectangle A

Rectangle C and Square D each have a perimeter of 60 cm. Again, the width of Rectangle C is 4 cm less than the width of Square D.

Square D

Rectangle C

(**Figures are not drawn to scale**)

Ignoring unit labels, find the sum of the perimeter of Rectangle A and the Area of Rectangle C.

a) 273   b) 84   c) 204   d) 157   e) 261

13) A ship sails from point A to point D. Along the way the ship stops at point B and point C. If the distance from A to B is $\frac{3}{4}$ of the total distance, and the distance from B to C is $\frac{1}{9}$ of the total distance then how far did the ship travel if the distance from C to D is 10 kilometers?

a) 72   b) 108   c) 45   d) 36   e) 56

14) If $xyz = 4$ and $y^2z = 5$, what is the value of $\frac{x}{y}$?

a) 20   b) 10   c) 1.25   d) 1   e) 0.8

15) Using the graph below, find the math exam score of any student with a final GPA in the 80's.
    Subtract this score from the median exam score for this data.
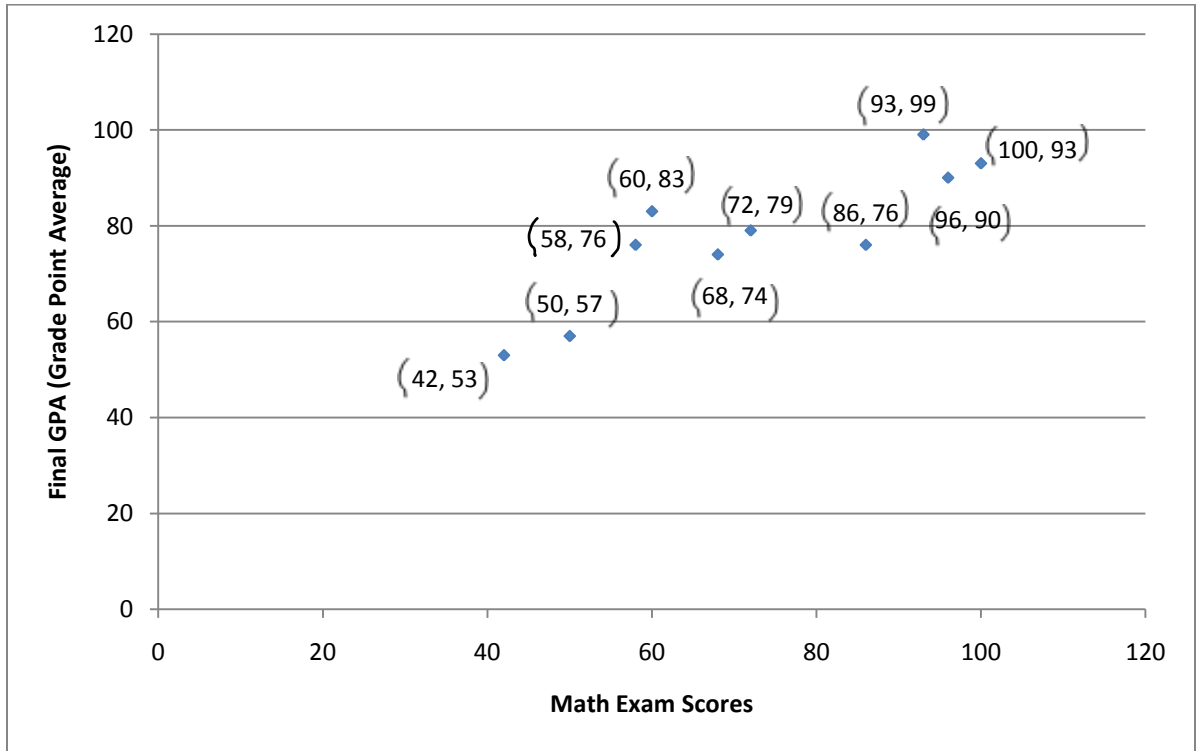
a) 2   b) -8.5   c) 8   d) 10   e) can't determine

16)   If the area of right triangle ABC is equal to $5a$, then $b = $?
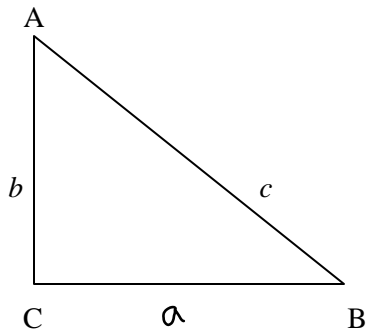
a) $\dfrac{5}{4}$ b) $\dfrac{5}{2}$ c) 5   d) 10   e) 20

17) There are 11 tulips in a bunch. Four are pink, three are white, three are yellow, and one is purple. What is the probability that you will get a white or purple one when you pick one tulip from a bunch?

    a) 3:7   b) 4:11   c) 7:11   d) 8:11   e) 4:7

18) Sally, Frank, and Trudy each calculated the circumference of the circular swimming pool that had a radius of 28 feet. However, they each used a different approximation for pi. Sally used 3.14, Frank used $\frac{22}{7}$, and Trudy used $3\frac{3}{14}$. Find the positive difference between the largest value and the smallest value of their calculations for the circumference.

    a) 56   b) 58.24   c) 4   d) 4.16   e) 232.96

19) A cube with edge of length 4 is divided into 8 identical cubes. How much greater is the combined surface area of the 8 smaller cubes than the surface area of the original cube?

    a) 48   b) 56   c) 96   d) 288   e) 384

20) A car travels a certain distance at 20 miles per hour (mph) and returns the same distance at 30 mph. What is the average speed for the entire round trip?

    a) 20 mph   b) 22 mph   c) 24 mph   d) 25 mph   e) 30 mph

21) How many times during a day (24 hours) will the hour and minute hands of a clock coincide?

    a) 11   b) 12   c) 22   d) 24   e) 48

22) If $x + y = 7$ and $x^2 + y^2 = 37$ then what is the value of xy ?

    a) 6   b) 12   c) 15   d) 22   e) 30

23) When a 3-digit integer is multiplied by another 3-digit integer the answer may have:

    a) 3-digits   b) 4-digits   c) 6-digits   d) 7-digits   e) 9-digits

24) The second hand of a clock is 5 cm long. How far will the tip of the second hand travel in one hour?

a) $36000\pi\ cm$ b) $144000\pi\ cm$ c) $3600\pi\ cm$ d) $600\pi\ cm$ e) $10\pi\ cm$

25) Find the missing number: $\dfrac{2}{3} = \sqrt{\dfrac{20}{?}}$

a) 9   b) 30   c) 36   d) 45   e) 900

26) Using the polynomial

$$\mathbf{a + ax + ax^{-2} + ax^{3} + ax^{-4} + \ldots + ax^{-8}.}$$

If $\mathbf{a = 8}$ and $\mathbf{x = \frac{1}{2}}$, find the value of the polynomial. Give your answer as a decimal.

27) Mrs. Smith is taking her students on a field trip and has a large van and a small bus to use to transport the students. The large van will hold 12 students and the small bus will hold 16 students. In order to decide who will ride in the van, she has her 28 students form a circle and count off in order 1, 2, 3, etc. to 28. Then she has them count off, starting with the first person, from 1 to 9 and every ninth person drops out (and gets in the van) until only 16 people are left. What number was originally assigned to the last student who got in the van?

28) A perfect number is a number which is the sum of all of its factors, except itself.

      6 is a perfect number since 1 + 2 + 3 = 6

An excessive number is a number which is greater than the sum of all of its factors, except itself.

A defective number is a number which is not perfect nor excessive.

      8 is an excessive number and 12 is a defective number.

Find the average of the largest perfect, largest excessive, and largest defective numbers less than 35. Give an exact answer. Do not round.

29) The points A (2, -1), B (-2, 2), and C (2, 14) are plotted and segments are drawn to form Triangle ABC. Ignoring unit labels, which is larger, the perimeter or the area of Triangle ABC? And by how much? Give an exact answer. Do not round.

30) Jane's calculator has a special * Key that obeys two rules;

#1: If the display shows a one-digit number, pressing the * Key replaces the display with twice the value.

#2: If the display shows a two-digit number, pressing the * Key replaces the display with the sum of the 2 digits.

If Jane enters the value 1 on her calculator and then presses the * key repeatedly, what will the display show after she presses the * key 50 times?